Period 4, March 6, 2025

vector quantities, Or Vectors

The magnitude is 100 pounds and the direction is 30° from the horizontal.

You are driving due north at 50 miles per hour. The magnitude is the speed, 50 miles per hour. The direction of motion is due north.

Directed Line Segments and Geometric Vectors

A line segment to which a direction has been assigned is called a **directed line segment**. **Figure 6.48** shows a directed line segment from P to Q. We call P the **initial point** and Q the **terminal point**. We denote this directed line segment by

$$\overrightarrow{PQ}.$$

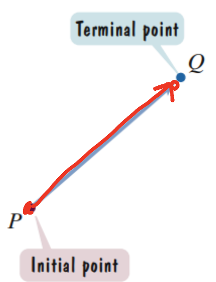
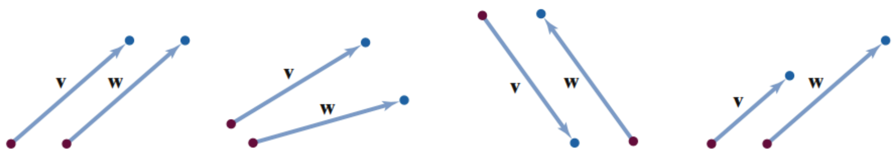


Figure 6.48 A directed line segment from P to Q

	Representing Vectors in Print	Representing Vectors on Paper		Representing Vectors in Print	Representing Vectors on Paper
Vector v	$\mathbf{v}$	$\vec{v}$	Magnitude of v	$\|\mathbf{v}\|$	$\|\vec{v}\|$
Vector w	$\mathbf{w}$	$\vec{w}$	Magnitude of w	$\|\mathbf{w}\|$	$\|\vec{w}\|$

Figure 6.49 shows four possible relationships between vectors **v** and **w**. In **Figure 6.49(a)**, the vectors have the same magnitude and the same direction, and are said to be *equal*. In general, vectors **v** and **w** are **equal** if they have the *same magnitude* and the *same direction*. We write this as **v** = **w**.



(a) **v** = **w** because the vectors have the same magnitude and same direction.

(b) Vectors **v** and **w** have the same magnitude, but different directions.

(c) Vectors **v** and **w** have the same magnitude, but opposite directions.

(d) Vectors **v** and **w** have the same direction, but different magnitudes.

Use **Figure 6.50** to show that **u** = **v**.

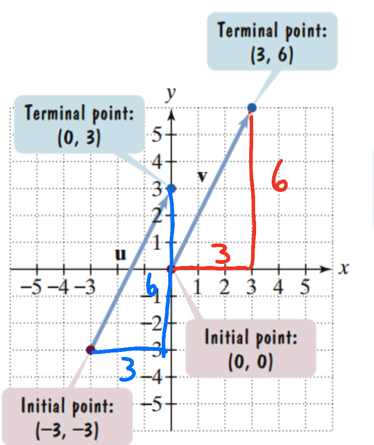


Figure 6.50

$\|U\| = \sqrt{3^2+6^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$

$\|V\| = \sqrt{3^2+6^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$

Slope of $U = \frac{\text{Rise}}{\text{Run}} = \frac{6}{3} = 2$

Slope of $V = \frac{\text{Rise}}{\text{Run}} = \frac{6}{3} = 2$

$V = 3i + 6j$

Scalar Multiplication

If k is a real number and **v** a vector, the vector $k\mathbf{v}$ is called a **scalar multiple** of the vector **v**. The magnitude and direction of $k\mathbf{v}$ are given as follows:

The vector $k\mathbf{v}$ has a *magnitude* of $|k|\,\|\mathbf{v}\|$. We describe this as the absolute value of k times the magnitude of vector **v**.

The vector $k\mathbf{v}$ has a *direction* that is

- the same as the direction of **v** if $k > 0$, and
- opposite the direction of **v** if $k < 0$.

• oppo

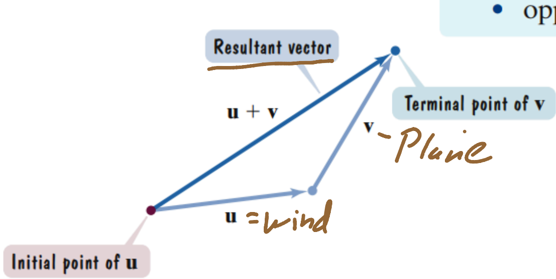
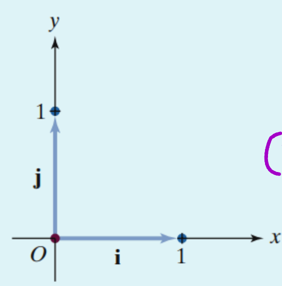
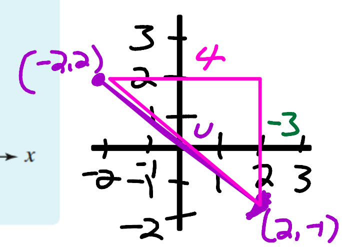


Figure 6.53 Vector addition $\mathbf{u} + \mathbf{v}$; the terminal point of **u** coincides with the initial point of **v**.

The i and j Unit Vectors

Vector **i** is the unit vector whose direction is along the positive x-axis. Vector **j** is the unit vector whose direction is along the positive y-axis.





$v = 4i - 3j$

Representing Vectors in Rectangular Coordinates

Vector **v** with initial point $P_1 = (x_1, y_1)$ and terminal point $P_2 = (x_2, y_2)$ is equal to the position vector

$$\mathbf{v} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j}.$$ ⇒ $V = (7-5)i + (1-3)j = 2i - 2j$

initial point of vector $V = (5, 3)$

Terminal point of vector $V = (7, 1)$

Adding and Subtracting Vectors in Terms of i and j

If $\mathbf{v} = a_1\mathbf{i} + b_1\mathbf{j}$ and $\mathbf{w} = a_2\mathbf{i} + b_2\mathbf{j}$, then

$$\mathbf{v} + \mathbf{w} = (a_1 + a_2)\mathbf{i} + (b_1 + b_2)\mathbf{j}$$

$$\mathbf{v} - \mathbf{w} = (a_1 - a_2)\mathbf{i} + (b_1 - b_2)\mathbf{j}.$$

If $\mathbf{v} = 5\mathbf{i} + 4\mathbf{j}$ and $\mathbf{w} = 6\mathbf{i} - 9\mathbf{j}$, find each of the following vectors:

a. $\mathbf{v} + \mathbf{w}$

$5i+4j+6i-9j$

$11i-5j$

b. $\mathbf{v} - \mathbf{w}$.

$(5i+4j)-(6i-9j)$

$5i+4j-6i+9j$

$-i+13j$

Scalar Multiplication with a Vector in Terms of i and j

If $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$ and k is a real number, then the scalar multiplication of the vector $\mathbf{v}$ and the scalar k is

$$k\mathbf{v} = (ka)\mathbf{i} + (kb)\mathbf{j}.$$

If $\mathbf{v} = 5\mathbf{i} + 4\mathbf{j}$, find each of the following vectors:

a. $6\mathbf{v}$

$6(5i+4j)$

$30i+24j$

b. $-3\mathbf{v}$.

$-3(5i+4j)$

$-15i+-12j$

If $\mathbf{v} = 5\mathbf{i} + 4\mathbf{j}$ and $\mathbf{w} = 6\mathbf{i} - 9\mathbf{j}$, find $4\mathbf{v} - 2\mathbf{w}$.

$4(5i+4j)-2(6i-9j)$

$20i+16j-12i+18j$

$8i+34j$

The Zero Vector

The vector whose magnitude is 0 is called the **zero vector, 0**. The zero vector is assigned no direction. It can be expressed in terms of **i** and **j** using

$$\mathbf{0} = 0\mathbf{i} + 0\mathbf{j}.$$

Properties of Vector Addition and Scalar Multiplication

If **u**, **v**, and **w** are vectors, and *c* and *d* are scalars, then the following properties are true.

Vector Addition Properties

1. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$	Commutative property
2. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$	Associative property
3. $\mathbf{u} + \mathbf{0} = \mathbf{0} + \mathbf{u} = \mathbf{u}$	Additive identity
4. $\mathbf{u} + (-\mathbf{u}) = (-\mathbf{u}) + \mathbf{u} = \mathbf{0}$	Additive inverse

Scalar Multiplication Properties

1. $(cd)\mathbf{u} = c(d\mathbf{u})$	Associative property
2. $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$	Distributive property
3. $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$	Distributive property
4. $1\mathbf{u} = \mathbf{u}$	Multiplicative identity
5. $0\mathbf{u} = \mathbf{0}$	Multiplication property of zero
6. $\|c\mathbf{v}\| = \lvert c\rvert\,\|\mathbf{v}\|$	Magnitude property

Finding the Unit Vector That Has the Same Direction as a Given Nonzero Vector v

For any nonzero vector **v**, the vector

$$\frac{\mathbf{v}}{\|\mathbf{v}\|}$$

is the unit vector that has the same direction as **v**. To find this vector, divide **v** by its magnitude.

Find the unit vector in the same direction as $\mathbf{v} = 5\mathbf{i} - 12\mathbf{j}$. Then verify that the vector has magnitude 1.

$\|V\| = \sqrt{(5)^2 + (-12)^2} = \sqrt{25+144} = \sqrt{169} = 13$

Unit vector of $V = \dfrac{5i - 12j}{13} = \dfrac{5}{13}i - \dfrac{12}{13}j$

Mag = 1

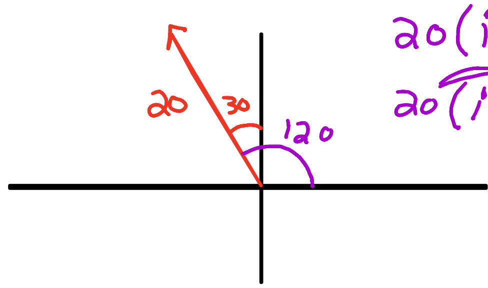
$\sqrt{\left(\frac{5}{13}\right)^2 + \left(\frac{-12}{13}\right)^2} = \sqrt{\frac{25}{169} + \frac{144}{169}} = \sqrt{\frac{169}{169}} = \sqrt{1} = 1$

Writing a Vector in Terms of Its Magnitude and Direction

Let $\mathbf{v}$ be a nonzero vector. If θ is the direction angle measured from the positive x-axis to $\mathbf{v}$, then the vector can be expressed in terms of its magnitude and direction angle as

$$\mathbf{v} = \|\mathbf{v}\| \cos\theta\mathbf{i} + \|\mathbf{v}\| \sin\theta\mathbf{j}.$$

The wind is blowing at 20 miles per hour in the direction N30°W. Express its velocity as a vector $\mathbf{v}$ in terms of $\mathbf{i}$ and $\mathbf{j}$.



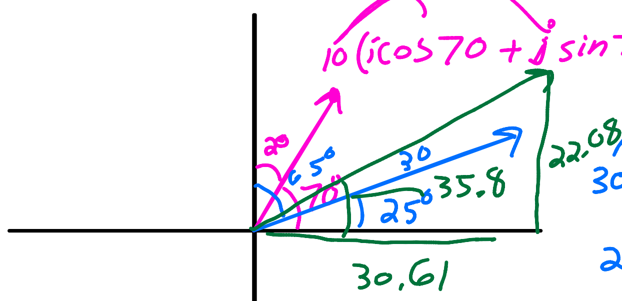
$20(i\cos 120 + j\sin 120)$

$20\left(i \cdot -\frac{1}{2} + j \cdot \frac{\sqrt{3}}{2}\right) = 20 \cdot -\frac{1}{2}i + 20 \cdot \frac{\sqrt{3}}{2}j$

$V = -10i + 10\sqrt{3}j$

Two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$, of magnitude 10 and 30 pounds, respectively, act on an object. The direction of $\mathbf{F}_1$ is N20°E and the direction of $\mathbf{F}_2$ is N65°E. Find the magnitude and direction of the resultant force. Express the magnitude to the nearest hundredth of a pound and the direction angle to the nearest tenth of a degree.

$10(\mathbf{i}\cos 70 + \mathbf{j}\sin 70) = 3.42\mathbf{i} + 9.4\mathbf{j}$



$30(\mathbf{i}\cos 25 + \mathbf{j}\sin 25)$

$27.19\mathbf{i} + \mathbf{j}\,12.68$

$\tan^{-1}\dfrac{22.08}{30.61} = 35.8°$

Result

$3.42\mathbf{i} + 9.4\mathbf{j} + 27.19\mathbf{i} + 12.68\mathbf{j}$

$30.61\mathbf{i} + 22.08\mathbf{j} = F_1 + F_2$

$\sqrt{30.61^2 + 22.08^2} = 37.74$

$F_1 + F_2$

Mag 37.74 pounds

N(90-35.8)E

N(54.2)E

What Brings to Equal

$-30.61\mathbf{i} - 22.08\mathbf{j}$

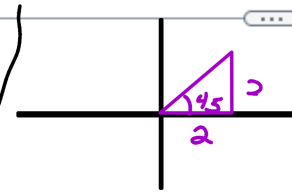
Use DeMoivre's Theorem to find the indicated power of the complex number. Write your answer in rectangular form.

$(2+2i)^3 = [2\sqrt{2}(\cos 45 + i \sin 45)]^3 = 2^3 \cdot \sqrt{2}^3 (\cos 3 \cdot 45 + i \sin 3 \cdot 45)$

The answer is $-16+16i$.

(Type your answer in the form a + bi.)

$\sqrt{2^2+2^2} = \sqrt{8} = 2\sqrt{2}$



$8 \cdot 2\sqrt{2}(\cos 135 + i \sin 135)$

$16\sqrt{2}\left(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$

$-\frac{16\sqrt{2}\cdot\sqrt{2}}{2} + i\frac{16\sqrt{2}\cdot\sqrt{2}}{2}$

$-16+16i$

Find $(1+i)^8$ using DeMoivre's Theorem. Write the answer in rectangular form, $a+bi$.

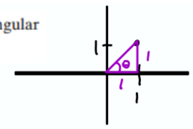
$r = \sqrt{1^2+1^2} = \sqrt{2}$

$(1+i)^8 = [\sqrt{2}(\cos 45° + i \sin 45°)]^8$

$= (\sqrt{2})^8[\cos 8(45) + i \sin 8(45)]$

$= 16(\cos 360 + i \sin 360)$

$16(1+i(0)) = 16(1+0) = 16$



$\tan\theta = \frac{1}{1}$

$\tan^{-1} 1 = 45° = \frac{\pi}{4}$

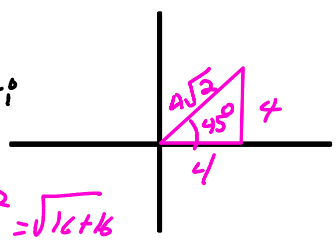
Find the quotient $\frac{z_1}{z_2}$ of the complex numbers. Leave your answer in polar form.

$z_1 = 4+4i \qquad z_2 = 5+5i$

$\frac{z_1}{z_2} = \frac{4}{5}(\cos 0 + i \sin 0)$

(Simplify your answer. Use integers or fractions for any numbers in the expression. Type any angles in radians between 0 and 2π. Express complex numbers in terms of i.)

$4+4i$



$\tan^{-1}\frac{4}{4} = 45°, \frac{\pi}{4}$

$\sqrt{4^2+4^2} = \sqrt{16+16}$

$\sqrt{32} = \sqrt{16 \cdot 2} = 4\sqrt{2}$

$z_1 = 4\sqrt{2}(\cos 45 + i \sin 45)$

$z_2 = 5+5i$

$|z| = 5\sqrt{2}$

$\tan^{-1}\frac{5}{5} = \tan^{-1} 1 = 45°$

$z_2 = 5\sqrt{2}(\cos 45 + i \sin 45)$

$\frac{z_1}{z_2} = \frac{4\sqrt{2}(\cos 45 + i \sin 45)}{5\sqrt{2}(\cos 45 + i \sin 45)} = \frac{4}{5}(\cos(45-45) + i \sin(45-45))$

$\frac{4}{5}(\cos 0 + i \sin 0)$

$\frac{4}{5}(1 + i \cdot 0) = \frac{4}{5}$

Convert to polar form and then perform the indicated operations. Express the answer in polar and rectangular form. Express the argument as an angle between 0° and 360°.

$i(3-3i)(-1+i\sqrt{3})$ = 1(cos 90 + i sin 90) · 3√2 (cos 315 + i sin 315) · 2 (cos 120 + i sin 120)

Type the answer in polar form.

$i(3-3i)(-1+i\sqrt{3}) = 6\sqrt{2}(\cos 165° + i \sin 165°)$ = 1·3√2·2 (cos(90+315+120) + i sin (90+315+120))   525   525

(Type an exact answer in the first answer box. Type all degree measures rounded to one decimal place as needed.)

Type the answer in rectangular form.

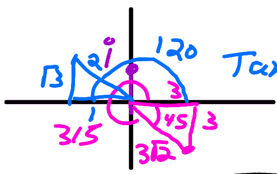
$i(3-3i)(-1+i\sqrt{3}) = -8.196 + 2.196i$

(Use integers or decimals for any numbers in the expression. Round to three decimal places as needed.)

525 − 360 = 165

= 6√2 (cos 165 + i sin 165)

→ 6√2 (−0.9659258 + i 0.258819)



Tan⁻¹ √3/1 = 60

DeMoivre's Theorem for Finding Complex Roots

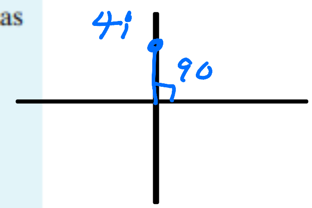
Let $w = r(\cos\theta + i\sin\theta)$ be a complex number in polar form. If $w \neq 0$, w has n distinct complex nth roots given by the formula

$$z_k = \sqrt[n]{r}\left[\cos\left(\frac{\theta + 2\pi k}{n}\right) + i\sin\left(\frac{\theta + 2\pi k}{n}\right)\right] \quad \text{(radians)}$$

$$\text{or} \quad z_k = \sqrt[n]{r}\left[\cos\left(\frac{\theta + 360°k}{n}\right) + i\sin\left(\frac{\theta + 360°k}{n}\right)\right] \text{(degrees)},$$

where $k = 0, 1, 2, \ldots, n-1$.

a² − b² = (a − b)(a + b)



Solve the following equation in the complex number system. Express solutions in both polar and rectangular form.

$x^4 - 256i = 0$ ⇒

4th Root n = 4

⁴√(x⁴) = ⁴√(256 i)

x = 4 ⁴√i

x⁴ − 256 = (x² − 16√i)(x² + 16√i)

0 = (x − 4⁴√i)(x + 4⁴√i)(x² + 16√i)

x = 4i

Write the solutions as complex numbers in polar form. Choose the correct answer below.

- A. 4(cos 22.5° + i sin 22.5°), 4(cos 247.5° + i sin 247.5°), 4(cos 202.5° + i sin 202.5°), 4(cos 337.5° + i sin 337.5°)
- B. 4(cos 67.5° + i sin 67.5°), 4(cos 157.5° + i sin 157.5°), 4(cos 247.5° + i sin 247.5°), 4(cos 337.5° + i sin 337.5°)
- C. 4(cos 67.5° + i sin 67.5°), 4(cos 157.5° + i sin 157.5°), 4(cos 202.5° + i sin 202.5°), 4(cos 292.5° + i sin 292.5°)
- ✔ D. 4(cos 22.5° + i sin 22.5°), 4(cos 112.5° + i sin 112.5°), 4(cos 202.5° + i sin 202.5°), 4(cos 292.5° + i sin 292.5°)

Write the solutions as complex numbers in rectangular form. Choose the correct answer below.

- A. − 1.5307i, 3.6955i, − 3.6955 − 1.5307i, − 1.5307 − 3.6955i
- B. 1.5307 − 3.6955i, − 1.5307i, 3.6955, 3.6955 + 1.5307i
- ✔ C. 3.6955 + 1.5307i, − 1.5307 + 3.6955i, − 3.6955 − 1.5307i, 1.5307 − 3.6955i
- D. − 1.5307 + 3.6955i, − 3.6955 − 1.5307i, − 1.5307 − 3.6955i, 3.6955 − 1.5307i

(4i)⁴ = 256 · i⁴

= 256 · i² · i²

= 256 · −1 · −1

= 256

$$4\left(\cos\frac{90+0\cdot 360}{4}+i\sin\frac{90+0\cdot 360}{4}\right)=4(\cos 22.5+i\sin 22.5)$$

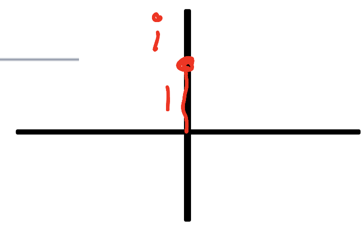
$$4\left(\cos\frac{90+1\cdot 360}{4}+i\sin\frac{90+1\cdot 360}{4}\right)=4(\cos 112.5+i\sin 112.5)$$

$$4\left(\cos\frac{90+2\cdot 360}{4}+i\sin\frac{90+2\cdot 360}{4}\right)=4(\cos 202.5+i\sin 202.5)$$

$$4\left(\cos\frac{90+3\cdot 360}{4}+i\sin\frac{90+3\cdot 360}{4}\right)=4(\cos 292.5+i\sin 292.5)$$

Find all the complex cube roots of i. Write the roots in rectangular form.

$n=3$



$z_0 = 0.9+0.5i$

(Type your answer in the form a + bi. Round to the nearest tenth.)

$z_1 = -0.9+0.5i$

(Type your answer in the form a + bi. Round to the nearest tenth.)

$z_2 = -i$

(Type your answer in the form a + bi. Round to the nearest tenth.)

$$z_0 = 1\left(\cos\frac{90+0\cdot 360}{3}+i\sin\frac{90+0\cdot 360}{3}\right)$$

$$1(\cos 30+i\sin 30)=1\left(\frac{\sqrt{3}}{2}+i\,\frac{1}{2}\right)=.866+0.5i=0.9+0.5i$$

$$z_1 = 1\left(\cos\frac{90+1\cdot 360}{3}+i\sin\frac{90+1\cdot 360}{3}\right)$$

$$1(\cos 150+i\sin 150)=1\left(-\frac{\sqrt{3}}{2}+i\,\frac{1}{2}\right)=(-.866+0.5i)=-0.9+0.5i$$

$$z_2 = 1\left(\cos\frac{90+2\cdot 360}{3}+i\sin\frac{90+2\cdot 360}{3}\right)$$

$$1(\cos 270+i\sin 270)=1(0+i(-1))=-i$$

Find all the complex fourth roots of $11 + 11i$. Write roots in rectangular form.

$\sqrt{11^2+11^2} = \sqrt{242}$

$11\sqrt{2}$

$\tan^{-1}\frac{11}{11} = 45$ $\quad 11 + 11i = $

$11\sqrt{2}(\cos 45 + i\sin 45)$

$z_0 = 1.9 + 0.4i$

(Type your answer in the form a + bi. Round to the nearest tenth as needed.)

$z_1 = -0.4 + 1.9i$

(Type your answer in the form a + bi. Round to the nearest tenth as needed.)

$z_2 = -1.9 - 0.4i$

(Type your answer in the form a + bi. Round to the nearest tenth as needed.)

$z_3 = 0.4 - 1.9i$

(Type your answer in the form a + bi. Round to the nearest tenth as needed.)

$\sqrt[4]{11\sqrt{2}}\left(\cos\frac{45+0\cdot 360}{4} + i\sin\frac{45+0\cdot 360}{4}\right) = \sqrt[4]{11\sqrt{2}}(\cos 11.25 + i\sin 11.25)$

$\sqrt[4]{11\sqrt{2}}\left(\cos\frac{45+1\cdot 360}{4} + i\sin\frac{45+1\cdot 360}{4}\right) = \sqrt[4]{11\sqrt{2}}(\cos 101.25 + i\sin 101.25)$

$\sqrt[4]{11\sqrt{2}}\left(\cos\frac{45+2\cdot 360}{4} + i\sin\frac{45+2\cdot 360}{4}\right) = \sqrt[4]{11\sqrt{2}}(\cos 191.25 + i\sin 191.25)$

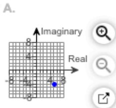
$\sqrt[4]{11\sqrt{2}}\left(\cos\frac{45+3\cdot 360}{4} + i\sin\frac{45+3\cdot 360}{4}\right) = \sqrt[4]{11\sqrt{2}}(\cos 281.25 + i\sin 281.25)$

Plot the complex number. Then write it in polar form.

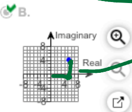
$3\sqrt{3} + 3i\sqrt{2}$ $\quad x = 3\sqrt{3}$ $\quad y = 3\sqrt{2}$

Plot the complex number $3\sqrt{3} + 3i\sqrt{2}$. Choose the correct graph below.

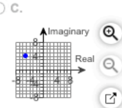
A.



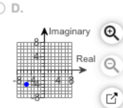
B.



C.



D.



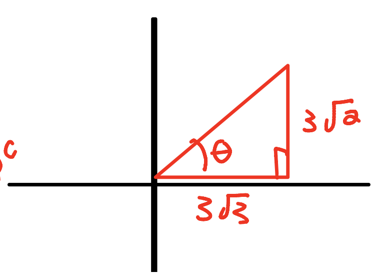
Write $3\sqrt{3} + 3i\sqrt{2}$ in polar form. Select the correct choice below and fill in the answer boxes to complete your choice.

(Type an exact answer in the first answer box. Simplify your answer. Type any angle measures in degrees, rounding to the nearest tenth as needed. Use angle measures greater than or equal to 0° and less than 360°.)

- A. $3\sqrt{5}(\cos 39.2^\circ + i\sin 39.2^\circ)$
- B. $\square(\cos \square^\circ + i\cos \square^\circ)$
- C. $\square(\sin \square^\circ + i\cos \square^\circ)$
- D. $\square(\sin \square^\circ + i\sin \square^\circ)$

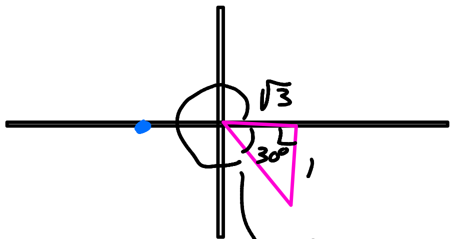
$\tan^{-1}\frac{3\sqrt{2}}{3\sqrt{3}}$

$\tan^{-1}\frac{\sqrt{2}}{\sqrt{3}} = 39.2°$



$\tan^{-1}\frac{1}{\sqrt{3}} = 30°$

$\sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$



$330 = 360 - 30 = -30°$

Use DeMoivre's Theorem to find the indicated power of the complex number. Write the answer in rectangular form.

$(\sqrt{3} - i)^6 = 2^6(\cos 6 \cdot -30 + i \sin 6(-30)) = 64(\cos -180 + i \sin -180)$

$64(-1 + i0) = -64$

$(\sqrt{3} - i)^6 = -64$

(Simplify your answer. Type an exact answer, using radicals and i as needed.)

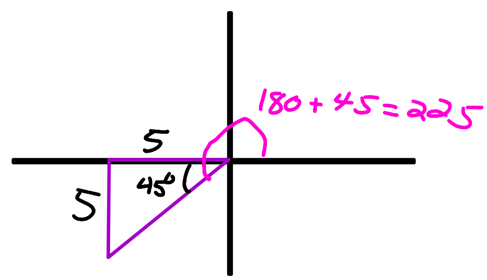
Plot the complex number. Then write the complex number in polar form. Express the argument in degrees.

$-5-5i$

Plot the complex number on the complex plane to the right.

Write the complex number $z=-5-5i$ in polar form. Select the correct choice below and fill in the answer box(es) within your choice.
(Type exact answers, using radicals as needed. Simplify your answers.)

- A. $z = 5\sqrt{2}\,(\cos 225^\circ + i\sin 225^\circ)$
- B. $z = \square(\sin \square^\circ + i\cos \square^\circ)$
- C. $z = \square(\sin \square^\circ + i\sin \square^\circ)$
- D. $z = \square(\cos \square^\circ + i\cos \square^\circ)$



$5\sqrt{2}(\cos 225 + i\sin 225)$

$\sqrt{5^2+5^2} = \sqrt{25+25} = \sqrt{50}$

$\sqrt{5\cdot 5\cdot 2} = 5\sqrt{2}$

Find all the complex square roots of $w = 36(\cos 30^\circ + i \sin 30^\circ)$. Write the roots in polar form with θ in degrees.

$n=2$

$z_0 = 6\left(\cos 15^\circ + i \sin 15^\circ\right)$

(Type answers in degrees. Simplify your answer.)

$z_1 = 6\left(\cos 195^\circ + i \sin 195^\circ\right)$

(Type answers in degrees. Simplify your answer.)

$$z_0 = \sqrt{36}\left(\cos \frac{30+0\cdot 360}{2} + i \sin \frac{30+0\cdot 360}{2}\right)$$

$$6(\cos 15 + i \sin 15)$$

$$z_1 = \sqrt{36}\left(\cos \frac{30+1\cdot 360}{2} + i \sin \frac{30+1\cdot 360}{2}\right)$$

$$= 6(\cos 195 + i \sin 195)$$